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Calibration of Cap/Floor Volatilities in a Global Discount Curve Environment

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Motivation

In the years after the financial crisis of 2008, the interest rate markets changed significantly. The most prominent process was the appearance of pronounced basis spreads between yield curves based on different tenors. In contrast to the time before the crisis, these spreads could not be ignored anymore, i.e. instead of one curve, several curves – according to the tenors the products considered were based on – had to be built and used in pricing algorithms. This situation has not switched back since then and is not expected to do so in the near future. Therefore, modern risk management and pricing has to incorporate the usage of multiple yield curves per currency.
Besides the yield curves, also the cap/floor volatility surfaces with respect to different tenors show a significant spread and thus have to be built separately. This is not a straight-forward task, because market data is not available for all tenors and all maturities. Instead, elaborated models have to be used to bootstrap, convert and smooth the volatilities that are quoted. This can be tackled with the help of the Microstep Volatility Engine (MVE). Details will be described elsewhere. The document at hand is focused on the question how to combine advanced multi-yield-curve approaches and the calibration of the cap/floor volatilities.

Yield Curve Building (basics)

Modern approaches to the calibration of yield curves consider the fact that OIS curves are commonly used for discounting. In addition to that, the question arises, what role the funding currency of an institution plays in this framework. This document refers to the so called "Global Discount Curve" method, as described in (Kenyon & Stamm, 2012). In that framework IBOR-based curves are built in the funding currency, starting from the OIS discount curve (e.g. in EUR: Euribor-1M, Euribor-3M, Euribor-6M and Euribor-12M with EONIA as discount curve). Using this data, discount curves in different currencies are built, using FX-forwards and cross-currency-swaps as benchmark instruments. The foreign discount curves are called XOIS-curves in the following. The last step comprises calibration of foreign forward curves (labeled XOyM-curves, where y stands for the underlying tenor).

Pricing of Caps/Floors

The calibration of cap/floor volatilities is based on the valuation of standard caps. These products are described by a start date T_0 (the fixing date of the first caplet), an end date T (the payment date of the last caplet), the tenor Δ and the strike K. The present value at time $t < T_0$ is given by

$$C^{\Delta,K}(t,T,\sigma_1,\dots,\sigma_n) = \sum_{i=1}^{n} c^{\Delta,K}(t,T_i,\sigma_i) \tag{1}$$

where $c^{\Delta,K}(t,T_i,\sigma_i)$ is the present value of a caplet with maturity T_i, strike K and tenor Δ.The parameters σ_i are forward caplet volatilities. Market quotes for theses instruments are implied par volatilities, i.e. the constant volatility Σ that replicates the present value of the cap with $\Sigma = \sigma_1 = \dots = \sigma_n$. The present value calculated with the help of the implied par volatility is denoted by $C^{\Delta,K}(t,T,\Sigma)$.

The value of a caplet $c^{\Delta,K}(t,T_i,\sigma_i)$ with nominal N is given by

$$c^{\Delta,K}(t,T_i,\sigma_i) = N\alpha(T_{i-1},T_i)P^d(t,T_i)[F^f\varphi(d_1) - K\varphi(d_2)]$$

Where $\alpha(T_{i-1},T_i)$ is the year fraction between T_{i-1} and T_i, $P^d(t,T_i)$ is the discount factor for time T_i as seen from time t, F^f is the forward rate and $\varphi(x)$ is the cumulative distribution function of the standard normal distribution. d_1 and d_2 are the usual coordinates as defined in Black's model for interest rate derivatives. For details see (Hull, 2003) or equivalent literature. The superscripts d and f refer to the curve that is used, i.e. discount and forward, respectively.

Note that the value of a caplet is comprised by a discount factor $P^d(t,T_i)$ that depends solely on the discount curve in the respective currency and an expected cashflow, that depends on the forward rate F^f which is extracted from the forward curve with tenor Δ. Therefore, calculation of the expected cashflow and discounting is done on different curves (within the same currency). The volatilities used in the pricing algorithm reflect the (market) implied dynamics of the forward volatility surfaces.

Calibration of Cap/Floor Volatilities in the Global Discount Curve Approach

As described above, the "Global Discount Curve" method involves the calibration of discount curves XOIS and forward curves XOyM in all currencies. The XOIS curves depend on FX-instruments between the currency under consideration and the domestic (funding) currency. Therefore, the foreign curves depend on the domestic currency, i.e. banks with different domestic currencies will build different curves for a given currency. That is a natural consequence of this approach. Banks will not agree on the valuation of certain products from counterparties using another domestic currency. This is also well known and described in Chapter 6.1 of (Kenyon & Stamm, 2012).

The market quotes for caps and floors in a given currency (as obtained through e.g. Bloomberg) are quoted as implied volatilities, where discounting is done via the local (for the respective currency) discount curve (OIS) and the forwards are obtained from the local forward curves. The curves do not necessarily match with the XOIS and XOyM curves for that respective currency. They rather correspond to curves as built by a bank using this currency as domestic one.

This situation gives rise to the question how to calibrate forward caplet volatilities. Which curves should be used and which assumptions should be applied to the bootstrapping process? In the following, we describe two possible methods. Subsequently, advantages and problems are described and assessed.

Method 1: Constant Cap Prices

In the first step of the calibration the quoted implied volatilities are converted to cap prices. These are the actual input data for the bootstrapping process. Of course, the prices initially have to be calculated using the standard market curves (e.g. IBOR forward curves and OIS curve for discounting, depending on the respective quotation).

Starting from these prices, the whole calibration is processed. The crucial point is, that during the calibration, the XOIS and XOyM curves will be used consistently. This means that we impose the condition that the cap prices have to be identical in the local currency setup and our "Global Discount Curve" approach.

Consequences

Since the discount curve XOIS and the local standard discount curve (OIS) do not match, the calibrated forward volatilities are different from the ones obtained in the local IBOR/OIS setup. Still, the cap prices of the market quoted caps are identical by construction of the volatilities.

Problems

The forward XOyM curves and the local IBOR forward curves are very similar. This has been shown empirically and is also quoted in (Kenyon & Stamm, 2012). Since the curves are almost identical for all times, they can be modeled as being driven by the same stochastic process. Forward rates obtained from the curves are identical and follow the same dynamics with the same volatility. Therefore, the above explained method for calibrating forward volatilities is questionable from a theoretical point of view. If used for calibrating a stochastic model (such as LMM), the effect of the adapted volatilities might be significant and is not counterbalanced by the discount curve (since this holds only for the standard caps used in the calibration process).

Secondly, the replication of market quotes is perfect for the standard quoted instruments – but what about non-standard instruments? When pricing a non-standard cap in a certain currency, the involved caplet volatilities are interpolated on the volatility surface given by the standard cap quotes. The calculated price should be as close as possible to the price seen by other market participants, where the ones that use the respective currency as domestic currency are of special interest (most probably they are very active counterparties in that market). Such a bank would use the volatility surface v^{OIS}, whereas we use the surface v^{XOIS}. These surfaces do not coincide. The surfaces v^{OIS} and v^{XOIS} are connected via a (analytically unknown) transformation $\mathbb{T}$. This transformation is implicitly done in our calibration and is non-linear (it involves an inversion of Black's formula and other non-trivial steps). Let now $\mathbb{I}(v)$ be a standard interpolation method on a volatility surface. Considering the non-linearity of the transformation $\mathbb{T}$, the following inequality will generally hold

$$\mathbb{I}(v^{XOIS}) = \mathbb{I}(\mathbb{T}(v^{OIS}) \neq \mathbb{T}(\mathbb{I}(v^{OIS}))$$

This means that – although we calibrated our surface in order to reproduce the standard market quotes – we do not know whether our prices for non-standard caps are close to market prices or not. The interpolation on the transformed surface can simply not (easily) be compared to the interpolation of the local OIS-surface. Still, the magnitude of the error introduced here cannot easily be estimated.

Method 2: Constant Caplet Volatilities

Comparison of the local IBOR forward curves and the XOyM forward curves obtained in the "Global Discount Curve" method shows that these entities are almost identical. This is an important finding, since it allows the conclusion that the underlying dynamics should also be nearly identical. Therefore, one can assume that the forward volatilities obtained from a calibration on the local IBOR and OIS curves can also be used for the XOyM curves – no additional calibration is needed.

Consequences

As stated above, the forward curves and the volatilities are identical in local IBOR/OIS and global XOyM/XOIS formulation when using this method. Since the discount curves OIS and XOIS certainly do show significant differences, the cap prices are not identical to the market quoted prices when evaluating within the "Global Discount Curve" method. This follows instantly when considering the above-mentioned pricing formula for caps. The expected cashflows will be identical, but the discount factors are different. Therefore, we will obtain a different price.

Problems

The above described consequence translates directly to the problem: We will not price any cap consistently with the market quotes.

Conclusion

Both methods – constant cap price and constant caplet volatility – have advantages and disadvantages. The first method allows a consistent pricing of standard quoted caps but imposes somewhat unnatural dynamics of the forward rates. Additionally, the accuracy of prices for non-standard caps cannot easily be estimated.

The second method is surprisingly simple, we just adopt the volatilities as calibrated in the standard process. Unfortunately, this means that all cap prices deviate from the standard curve approach because the discount curve XOIS is different.

Although the last point seems to disqualify the second method, it should still be considered because the deviation of the prices is very transparent. It just stems from the different discount factors. So, if interested in the market quoted price, one simply has to recalculate the price using the local OIS curve for the respective currency. This curve has to be present in the system anyway – it is required during the calibration process. Even better, the difference is given by the spread between the curves XOIS and OIS – we just have to add a spread-dependent discount factor in the valuation of the cap (the expected cashflows are identical!). Thus, the difference in price is very transparent – both easily understandable and quantifiable. The forward dynamics implied by the market is not altered at all.

The second method is also favorable when calculating risk figures. It assures that the reaction on volatility changes is consistent with the market view and does not introduce an additional dependency of volatility based risk on the basis spread. Still, the magnitude of the effects is not clear.

The usage of the "Global Discount Curve" method considers individual funding costs and collateral currency – and therefore causes market data to differ from other market participants' data. While the first method absorbs these differences in the volatilities and reproduces the market prices for standard caps and floors, but modifies the forward dynamics artificially, the second method shows deviations to the market, but these are transparent and can always be explained by the spread of the discount curves.

Technical Implications on the Microstep Volatility Engine (MVE)

The technical setup of the MVE supports both approaches. As quoted above, the method of constant cap prices is absorbed completely in the first step of the calibration. This can easily be done in our framework, without requiring a significant change of the code.

Suggestion of further steps

The choice of the method should be made diligently by considering the effects on pricing and risk figures. The magnitude and the scope of mispricing of non-standard products in the first method and for standard caps and floors in the second should be evaluated. This error has to be put in relation to the exposure of the bank in these product groups (market volume by currency and non-standard product catalogue).

Bibliography

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